EE 232 Lightwave Devices
Lecture 14: Quantum Well and Strained Quantum Well Laser

Reading: Chuang, Sec. 10.3-10.4
(There is also a good discussion in Coldren, Appendix 11)

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Quantum Well Gain

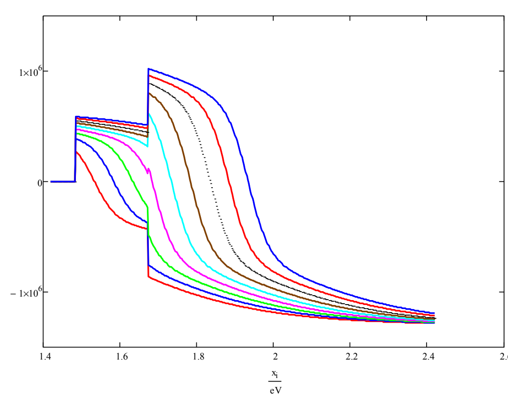
QW Material Gain:

$$g(\hbar\omega) = C_0 \left| \hat{e} \cdot \vec{P}_{cv} \right|^2 \rho_r^{2d}(\hbar\omega) f_g(\hbar\omega)$$

$$C_0 = \frac{\pi e^2}{n_r c \varepsilon_0 m_0^2 \omega}$$

$$\left| \hat{e} \cdot \vec{P}_{cv} \right|^2 \approx \frac{m_0}{6} E_p$$

$$\rho_r^{2d}(E) = \frac{m_r^*}{\pi \hbar^2 L_z} \sum_{n=1}^{\infty} H(E - E_{en})$$



Advantages of Quantum Well Lasers

(1) Low threshold current density:

Compare fundamental material property

$\rightarrow$ Transparency current density

$$J_{tr}^{bulk} = \frac{qN_{tr}^{bulk}}{\tau} d_{active}$$

$$J_{tr}^{QW} = \frac{qN_{tr}^{QW}}{\tau} L_z$$

Since $N_{tr}^{bulk} \approx N_{tr}^{QW} \Rightarrow \boxed{\frac{J_{tr}^{QW}}{J_{tr}^{bulk}} = \frac{L_z}{d_{active}}} \sim \frac{10\text{ nm}}{100\text{ nm}}$

`10%

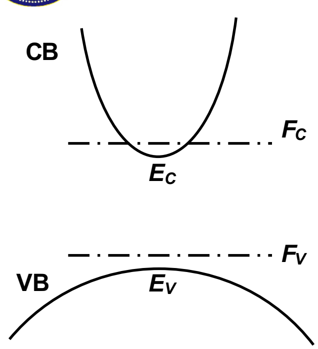
(2) Higher differential gain $\rightarrow$ Larger bandwidth:

Resonance frequency: $\omega_R = \sqrt{\frac{v_g a S}{\tau_p}} \propto \sqrt{a} = \sqrt{\frac{\partial g}{\partial N}}$

(3) Lower chirp:

Smaller wavelength shift when the laser is directly modulated

Transparency Carrier Concentration in Bulk



Transparency Condition
(Bernard-Duraffourg
Inversion Condition)

$\Delta F = F_C - F_V = E_g$

At transparency: $F_C - F_V = E_C - E_V$

or $F_C - E_C = F_V - E_V$

Let $\Delta = \frac{F_C - E_C}{k_B T} = \frac{F_V - E_V}{k_B T}$

Electron concentration: $\because F_C > E_C$

$$N = 2\left(\frac{\pi m_e^* k_B T}{2\pi^2 \hbar^2}\right)^{3/2} \frac{4}{3\sqrt{\pi}} \left(\frac{F_n - E_C}{k_B T}\right)^{3/2} = N_C \cdot \frac{4}{3\sqrt{\pi}} \Delta^{3/2}$$

Hole concentration: $\because F_V > E_{h1}$

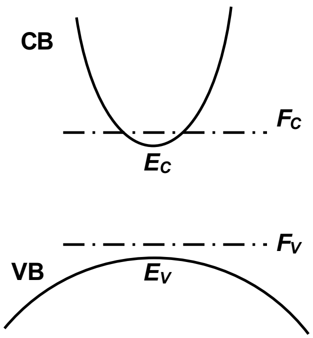
$$P = 2\left(\frac{\pi m_h^* k_B T}{2\pi^2 \hbar^2}\right)^{3/2} e^{-\frac{F_V - E_V}{k_B T}} = N_V e^{-\Delta}$$

$$N = P \Rightarrow \frac{4}{3\sqrt{\pi}} \Delta^{3/2} = \left(\frac{m_h^*}{m_e^*}\right)^{3/2} e^{-\Delta} \Rightarrow \text{Solve } \Delta$$

For GaAs ($m_e^* = 0.067 m_0, m_h^* = 0.5 m_0$)

$\Delta = 2.15, \quad N = 9 \times 10^{17}\ \text{cm}^{-3}$

Transparency Carrier Concentration in QW



Transparency Condition
(Bernard-Duraffourg Inversion Condition)

$$\Delta F = F_C - F_V = E_g$$

At transparency: $F_C - F_V = E_{e1} - E_{h1}$

or $F_C - E_{e1} = F_V - E_{h1}$

Let $\Delta = \dfrac{F_C - E_{e1}}{k_B T} = \dfrac{F_V - E_{h1}}{k_B T}$

Electron concentration: $\because F_C > E_{e1}$

$$N = \frac{m_e^* k_B T}{\pi \hbar^2 L_z}\left(\frac{F_C - E_{e1}}{k_B T}\right) = N_C^{2d} \cdot \Delta$$

Hole concentration: $\because F_V > E_{h1}$

$$P = \frac{m_h^* k_B T}{\pi \hbar^2 L_z} e^{-\frac{F_V - E_{h1}}{k_B T}} = N_V^{2d} e^{-\Delta}$$

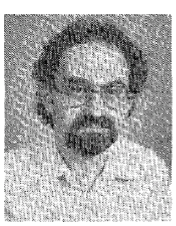
$$N = P \ \Rightarrow\ \Delta = \frac{m_h^*}{m_e^*} e^{-\Delta} \Rightarrow \text{Solve } \Delta$$

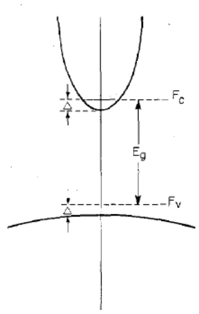
For GaAs ($m_e^* = 0.067 m_0, m_h^* = 0.5 m_0$)

$\Delta = 1.56,\ \ N = N_C^{2d} \cdot \Delta = 10^{18}\ \text{cm}^{-3}$

Note: N is independent of L_z

Reduction of Lasing Threshold Current Density by Lowering Valence Band Effective Mass

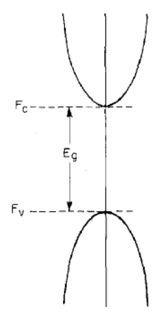




Ordinary Semiconductor

$m_h^* \approx 6 m_e^*$

High transparency carrier concentration



Ideal Semiconductor

$m_h^* \approx m_e^*$

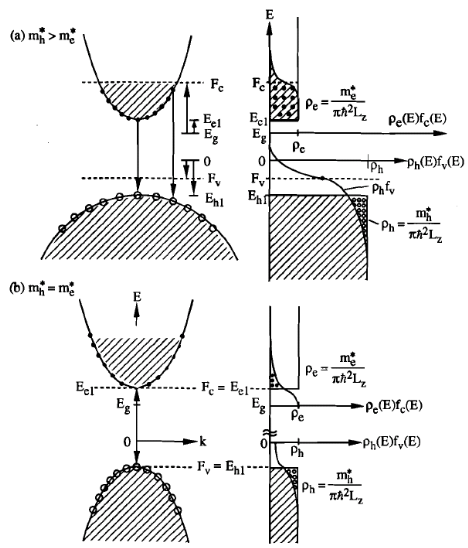
Low transparency carrier concentration

Bernard-Duraffourg Condition:

$$F_C - F_V \geq \hbar\omega \geq E_{e1} - E_{h1}$$

- Yablonovitch, E.; Kane, E., "Reduction of lasing threshold current density by the lowering of valence band effective mass," *Lightwave Technology, Journal of*, vol.4, no.5, pp. 504-506, May 1986
- Yablonovitch, E.; Kane, E.O., "Band structure engineering of semiconductor lasers for optical communications," *Lightwave Technology, Journal of*, vol.6, no.8, pp.1292-1299, Aug 1988

Bernard-Duraffourg Condition in Quantum Well



Bernard-Duraffourg Condition:

$F_C - F_V = E - E_{h1}$

(a) $m_h^* > m_e^*$ (as in most semiconductors)

$F_V > E_{h1}$

$F_C \gg E_{e1}$

$$N_{tr} = \rho_e^{2d}(F_C - E_{e1}) = \frac{m_e^*}{\pi\hbar^2 L_z}(F_C - E_{e1})$$

Large N_{tr} $\rightarrow$ High threshold current

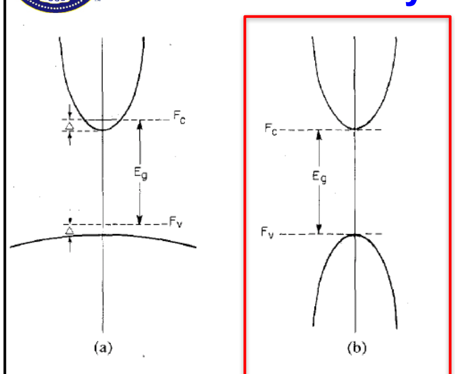
(b) $m_h^* = m_e^*$ (Ideal semiconductor)

$F_V = E_{h1}$

$F_C = E_{e1}$

$$N_{tr} = \frac{m_e^*}{\pi\hbar^2 L_z}\int_{E_{e1}}^{\infty} f_C(E)\,dE \quad \text{is low}$$

Transparency Carrier Concentration for Ordinary Semiconductor



Transparency Condition:

$F_C - F_V = E_{e1} - E_{h1}$

(b) Ideal Semiconductor

$$m_h^* = m_e^* \quad \Rightarrow \quad \boxed{F_V = E_{h1} \quad F_C = E_{e1}}$$

$$N_{tr} = \frac{m_e^*}{\pi \text{h}^2 L_z}\int_{E_{e1}}^{\infty} \frac{1}{1+e^{\frac{E-E_{e1}}{k_B T}}}dE$$

$$= \frac{k_B T m_e^*}{\pi \text{h}^2 L_z}\int_0^{\infty}\frac{1}{1+e^x}dx$$

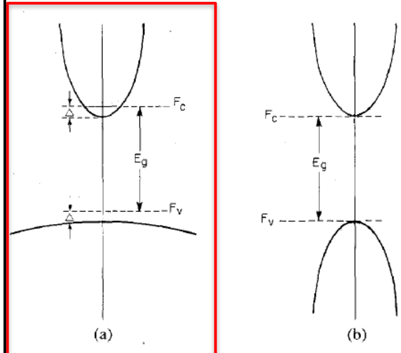
$$= \frac{k_B T m_e^*}{\pi \text{h}^2 L_z}\left(-\ln(1+e^{-x})\right)\Big|_0^{\infty}$$

$$= \frac{k_B T m_e^*}{\pi \text{h}^2 L_z}\ln 2$$

For m_e^*=0.067m_0

$N_{tr} \approx 4.6\times 10^{17}\,cm^{-3}$

Transparency Carrier Concentration for Ordinary Semiconductor



(a) Ordinary Semiconductor

$$N_{tr} = \rho_e^{2d}(F_C - E_{e1}) = \frac{m_e^*}{\pi\hbar^2 L_z}\Delta$$

To estimate Δ, note that $N = P$

$$P = N_V^{2d} e^{\frac{-\Delta}{k_B T}} = \frac{k_B T m_h^*}{\pi\hbar^2 L_z} e^{\frac{-\Delta}{k_B T}}$$

$$N = P \quad \Rightarrow \quad e^{\frac{-\Delta}{k_B T}} = \frac{\Delta}{k_B T}\frac{m_e^*}{m_h^*}$$

Transparency Condition:

$$F_C - F_V = E_{e1} - E_{h1}$$

For $m_h^* \approx 6m_e^*$ (in 1.55μm laser),

$\Delta = 1.43 k_B T$

$$N_{tr} = 1.43\frac{k_B T m_e^*}{\pi\hbar^2 L_z}$$

Effective Mass Asymmetry Penalty

$$\frac{N_{tr}^{Ordinary}}{N_{tr}^{Ideal}} = \frac{1.43}{\ln 2} = 2$$

Threshold current density reduction is more than a factor of 2:

$$J_{th} = J_{nonrad} + J_{rad} + J_{Auger}$$

$$\frac{J_{th}}{qd} = AN + BN^2 + CN^3 = \frac{N}{\tau} + BN^2 + CN^3$$

τ: Shockley-Read-Hall nonradiative recombination lifetime

J_{Auger} is greatly reduced when N is lowered

(1) N^3 is reduced by 8x

(2) C is also reduced due to band structure change by strain

Bandgap-vs-Lattice Constant of Common III-V Semiconductors

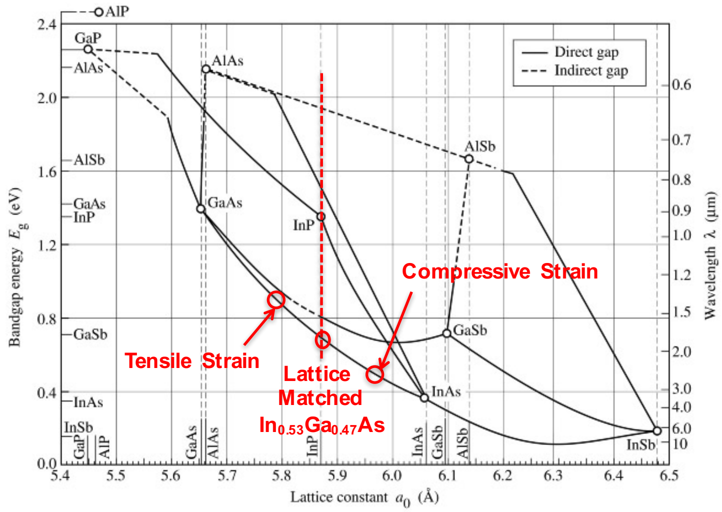
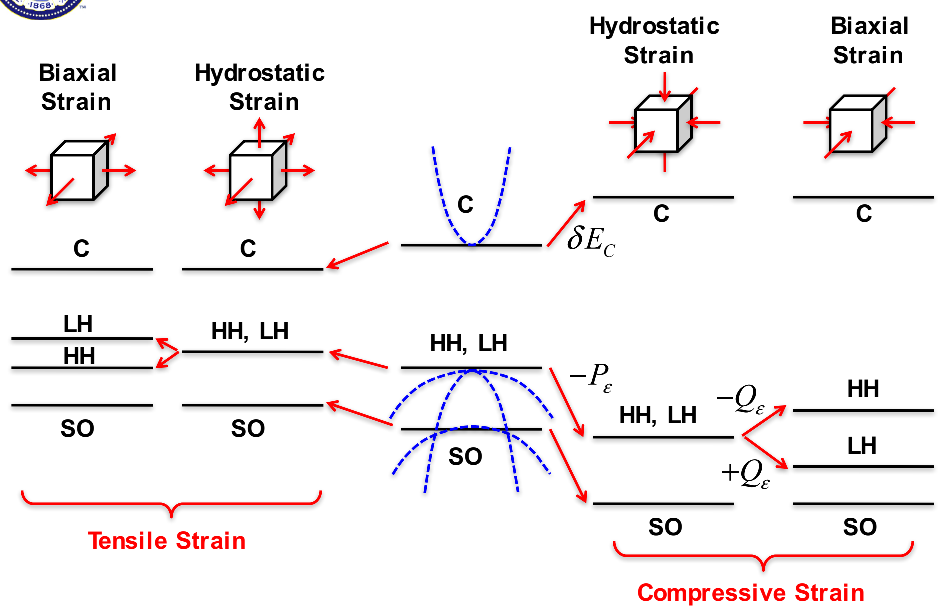


Fig. 7.6. Bandgap energy and lattice constant of various III-V semiconductors at room temperature (adopted from Tien, 1988).

Qualitative Band Energy Shifts Under Strain



Strain and Stress

$$\varepsilon = \varepsilon_{xx} = \varepsilon_{yy} = \frac{a_0 - a(x)}{a_0}$$

a_0 : lattice constant of InP

$$\begin{cases} \varepsilon < 0 : \text{ compressive strain} \\ \varepsilon > 0 : \text{ tensile strain} \end{cases}$$

$$\varepsilon_\perp = \varepsilon_{zz} = -2\frac{C_{12}}{C_{11}}\varepsilon$$

C_{ij} : Compliance Tensor

$C_{12} \approx 0.5 C_{11}$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} \\ C_{12} & C_{11} & C_{12} \\ C_{12} & C_{12} & C_{11} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \end{bmatrix}$$

Biaxial stress:

$\sigma_{xx} = \sigma_{yy} = \sigma$

$\sigma_{zz} = 0$

$\Rightarrow \quad C_{12}\varepsilon_{xx} + C_{12}\varepsilon_{yy} + C_{11}\varepsilon_{zz} = 0$

$$\varepsilon_{zz} = -2\frac{C_{12}}{C_{11}}\varepsilon$$

Band Edge Shift

$$E_C = E_g(x) + \delta E_C$$
$$E_{HH} = -P_\varepsilon - Q_\varepsilon$$
$$E_{LH} = -P_\varepsilon + Q_\varepsilon$$

$$\delta E_C = a_C(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) = 2a_C\left(1 - \frac{C_{12}}{C_{11}}\right)\varepsilon$$

$$P_\varepsilon = -a_V(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) = -2a_V\left(1 - \frac{C_{12}}{C_{11}}\right)\varepsilon$$

$$Q_\varepsilon = -b\left(\frac{\varepsilon_{xx} + \varepsilon_{yy}}{2} - \varepsilon_{zz}\right) = -b\left(1 + 2\frac{C_{12}}{C_{11}}\right)\varepsilon$$

$a = a_C - a_V$: hydrostatic potential

b : shear potential

Strain Parameters in III-V

(Coldren, p.535)

TABLE A11.1 Strain Parameters in III–V Semiconductors.

	Lattice Constant	Deformation Potentials (eV)			Elastic Moduli (10^{11} dyn/cm^2)			(10^{-6} eV/bar)	
Material	a(Å)	a	b	d	C_{11}	C_{12}	C_{44}	dE/dP	Δ(eV)
GaAs	5.6533	−8.68	−1.7	−4.55	11.88	5.38	5.94	11.5	0.34
InAs	6.0583	−5.79	−1.8	−3.6	8.329	4.526	3.959	10.0	0.371
AlAs*	5.6611	−7.96	−1.5	−3.4	12.02	5.70	5.89	10.2	0.30
GaP*	5.4512	−9.76	−1.5	−4.6	14.12	6.253	7.047	11.0	0.10
InP	5.8688	−6.16	−2.0	−5.0	10.22	5.76	4.60	8.5	0.10
AlP*	5.4635	−8.38	−1.75	−4.8	13.2	6.3	6.15	9.75	0.10
GaSb	6.0959	−8.28	−1.8	−4.6	8.842	4.026	4.322	14.7	0.8
InSb	6.4794	−7.57	−2.0	−4.8	6.47	3.65	3.02	16.5	0.98
AlSb*	6.1355	2.04	−1.35	−4.3	8.769	4.341	4.076	−3.5	0.75

* Indirect gap.

Band-Edge Profile and Subband Dispersion